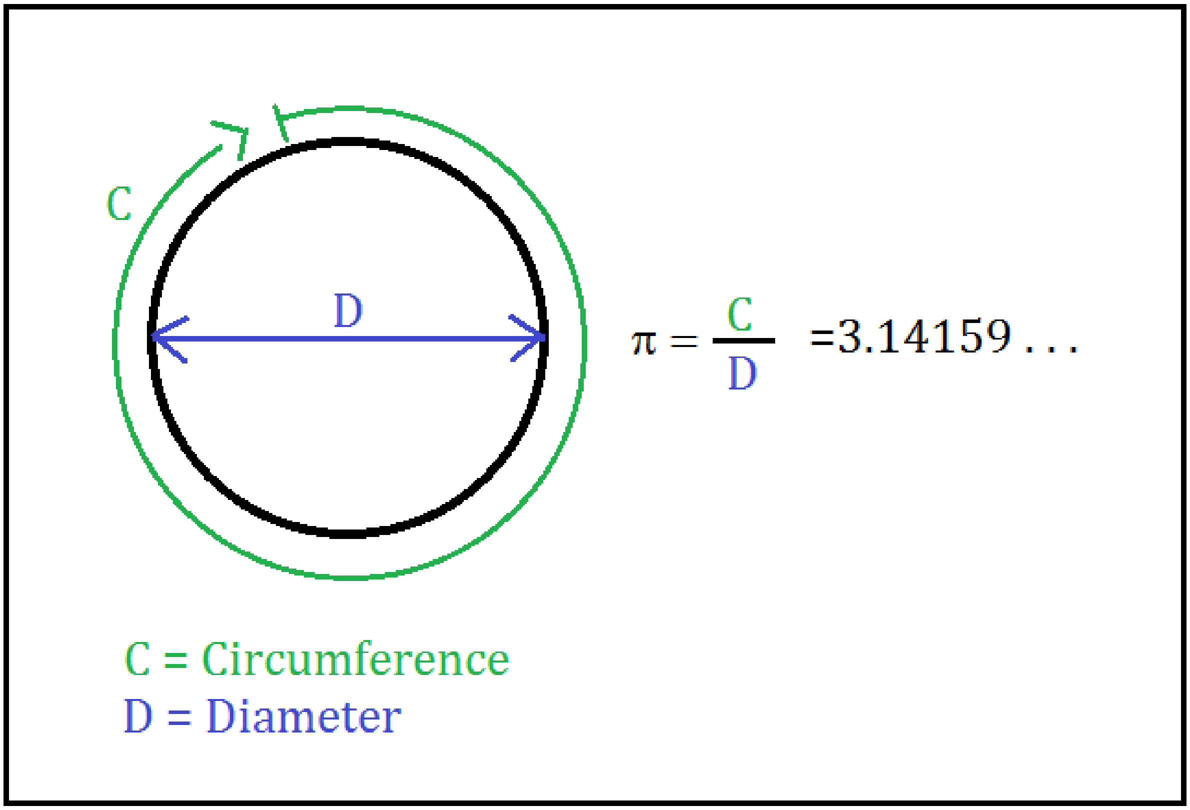
BUFFON'S TOOTHPICK

EXPERIMENTALLY ESTIMATING PI

π is an important physical constant. It first appeared when determining the ratio between a circle's circumference and it's diameter.



But it appears in other places too, even places where there appears to be no circles (though there is always some type of circular aspect involved)

$$e^{i\,\pi} + 1 = 0$$

The mathematical identity known as Euler's Formula

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

Heisenberg's Uncertainty Principle

$$\vec{E} = \frac{Q}{4\pi\varepsilon_o r^2}\hat{r}$$

Equation for Electric Field

π is a feature of the universe. We can experimentally determine pi using a set of ten toothpicks and a sheet of paper.

The method of determining pi in this manner was first proposed by the Comte de Buffon in the eighteenth century. The method used to calculate the probability is quite complex, so we'll just look at the end result

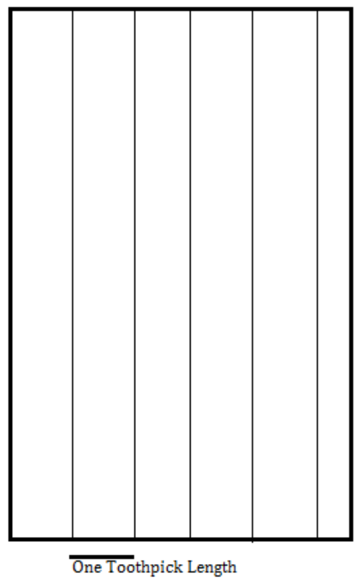
$$\pi \approx \frac{2 \times Number\ Dropped}{Number\ that\ Cross\ a\ Line}$$

By tossing toothpicks, and counting how many end up crossing a line, we can experimentally determine π!

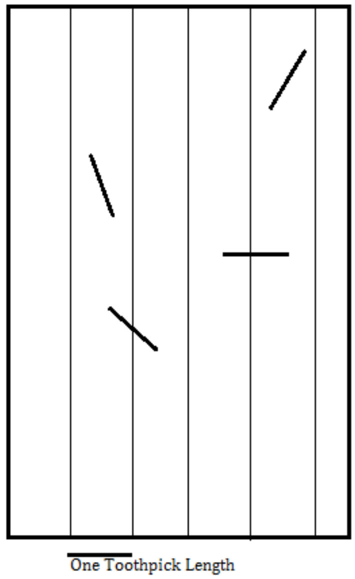
Because this procedure is taking advantage of random statistics, our result won't be perfect, but it should be close. The more toothpicks you drop, the closer and closer you should get to the actual value!

PROCEDURE

Measure the length of one toothpick. Record it. Then, on a sheet of paper, draw a set of vertical lines (covering the entire length of the paper) that are the length of your toothpick apart. It will look like this:



Drop the ten toothpicks one at a time. If they fall off the paper, that's okay! Just drop them again. Once all ten toothpicks are dropped, count how many are crossing a line. Record the results.



Repeat the procedure until you have dropped a total of 100 toothpicks. Record the results. Then use Buffon's formula to find your value of π. How close did you get? Continue dropping toothpicks to get a better value of π. Try for a thousand drops!

# of Drops = ___________________

Crossing a Line = __________________

π = ___________________

CALCUTAING PI (ADVANCED)

If you want to try something a little more advanced, we can actually calculate π to any desired accuracy using the following formula:

$$\pi = 4 \times \sum_{n=1}^{\infty} \frac{-1^{n+1}}{2n-1}$$

This is an infinite series. But we can approximate π by only calculating a few of the terms.

This is how it works. We start with n=1. Substitute 1 for n:

$$4 \times \frac{-1^{1+1}}{2(1)-1} =$$

$$= 4 \times \frac{-1^{2}}{1}$$

-1 to any even power is equal to 1. -1 to any odd power is -1

$$= 4 \times \frac{1}{1} = 4$$

Obviously, π does not equal 4. So we have to go to the next term, this time using n=2

$$4 \times \frac{-1^{2+1}}{2(2)-1} = 4 \times \frac{-1^{3}}{4-1} = 4 \times \frac{-1}{3} = -\frac{4}{3}$$

We then add this to our previous term

$$\pi = 4 - \frac{4}{3} = 2.7$$

This is not really close either, but it is closer to 3.14 than 4 was. To get closer and closer to the real value, continue adding more and more terms. I've completed the first five terms here.

$$\pi = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} = 3.3$$

Notice the pattern: alternating plusses and minuses, and the denominator increases by 2 each time. Continuing on to include ten terms, just going by the pattern now:

$$\pi = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \frac{4}{13} - \frac{4}{15} + \frac{4}{17} - \frac{4}{19} = 3.0$$

We're still not there, but we're getting close. Continue calculating terms until you get 3.1. How many will it take you?